where ΔP is the indicated pressure differential due to the difference in mercury levels in the mercury reservoir and receiver, given by

$$\Delta P = \frac{g(\Delta h)(\rho_m - \rho_f)}{g_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If the cross-sectional areas of the two vessels are equal,

$$- A_1 \frac{dh_1}{d\theta} = A_2 \frac{dh_2}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or, the decrease in mercury level in the reservoir is equal to the increase in level in the receiver. This means that

$$\frac{d(\Delta h)}{d\theta} = 2 \frac{dh_1}{d\theta} = - 2 \frac{dh_2}{d\theta}, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and that the difference between initial and final mercury differential will be equal to twice the electrode spacing,

$$\Delta h_2 - \Delta h_1 = - 2\varepsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

At any given time, θ, the general equation representing steady flow in the viscometer can be written,

$$\Delta P = a(\Delta h) = b_f Q_f + c_f Q_f^2 + b_m Q_m \, , \quad . \quad (11)$$

where $a = (\rho_m - \rho_f)g/g_c$, $b = 8\mu L/g\pi r^4$, and $c = \beta\rho/g\pi^2 r^4$.

Q is instantaneous flow rate at time, θ, and subscripts f and m refer to the fluid and mercury, respectively.

The instantaneous mercury flow rate, Q_m, can be expressed as a function of the area of the receiver, A, and the rate of change of h_2, $dh_2/d\theta$, as

$$Q_m = A \frac{dh_2}{d\theta} = - \frac{A}{2} \frac{d(\Delta h)}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If the fluid being displaced is incompressible,

$$Q_m = Q_f = - \frac{A}{2} \frac{d(\Delta h)}{d\theta}.$$

The assumption of an incompressible test fluid is valid at this point in the derivation, since only differential changes in mercury level are being considered.

The differential equation for the viscometer now becomes,

$$- a(\Delta h) = (b_f + b_m) \frac{A}{2} \frac{d(\Delta h)}{d\theta} + c_f \frac{A}{2} \left[\frac{d(\Delta h)}{d\theta}\right]^2 \quad . \quad . \quad . \quad (13)$$

Integration of the roots of Eq. 13 results in two equations, one of which gives a negative flow time and was therefore discarded. The second solution, with integration between the limits of time = 0 and θ, and Δh = Δh_1 and Δh_2,

$$\theta = - \frac{(b_f + b_m)}{2c_f} \left\{ \ln \frac{\Delta h_2}{\Delta h_1} + \propto_2 - \propto_1 + \ln \frac{(\propto_2 - 1)(\propto_1 + 1)}{(\propto_2 + 1)(\propto_1 - 1)} \right\}, \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where

$$\propto_1 = \sqrt{1 + \frac{8ac_f\Delta h_1}{A(b_f + b_m)^2}}$$

$$\propto_2 = \sqrt{1 + \frac{8ac_f\Delta h_2}{A(b_f + b_m)^2}}$$

This rigorous solution of the flow equation for the viscometer requires use of an iterative procedure to solve for the fluid viscosity.

To obtain a first approximation of the fluid viscosity to use in solving Eq. 14, Eq. 13 was solved assuming Langhaar's correction term to be constant and applicable to the integrated form of the modified equation. This resulted in an equation which may be solved directly for viscosity.

$$\mu_f = \frac{\pi g r_c^4 \theta}{8 L_c V_D} \left[\frac{\Delta h_{1m}(\rho_m - \rho_f) g}{g_c} - \frac{\beta \rho_f V_D^2}{g \pi r_c^4 \theta^2} \right] - \frac{L_m r_c^4 \mu_m}{r_m^4 L_c} \quad . \quad . \quad . \quad (15)$$

which has been termed a "pseudo-steady-state" equation. Both equations were programmed for solution by use of IGT's electronic digital computer, ALWAC III, and the calculated viscosity values for the two equations agree within 0.1 per cent. By using the viscosity value obtained by solution of the pseudo-steady-state equation as the first approximation in the rigorous equation, very rapid convergence is obtained.

EFFECT OF COMPRESSIBILITY ON VOLUME OF FLUID DISPLACED THROUGH THE CAPILLARY TUBE

For an incompressible fluid the volume contained in the receiver between the electrodes is the actual volume displaced through the capillary tube. However, since the pressure in the mercury receiver decreases slightly during the run, the fluid confined in the receiver and fittings will expand, resulting in a slightly larger volume being displaced through the capillary tube than the volume of mercury contained between the electrodes. The correction is derived as follows.

Total moles of gas initially in receiver when mercury level contacts the long electrode,

$$N_1 = \left(P_a + \frac{\Delta h_2}{2} + \varepsilon \right)(V_F + V_m)/zRT \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and moles of gas remaining when mercury contacts the short electrode,

$$N_2 = \left(P_a + \frac{\Delta h_2}{2} \right) V_F/zRT \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The moles displaced are, therefore,

$$N_D = N_1 - N_2 \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

or

$$P_a V_D = \left(P_a + \frac{\Delta h_2}{2} + \varepsilon \right)(V_F + V_m) - \left(P_a + \frac{\Delta h_2}{2} \right) V_F \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

then

$$V_D = V_m \left(1 + \frac{\Delta h_2 + 2\varepsilon}{2 P_a} \right) + \frac{\varepsilon V_F}{P_a} \quad . \quad . \quad (20)$$

For liquids, and for gases at pressures above 1,500 psia, the correction was less than 0.1 per cent and was therefore neglected in the calculations and V_m used as the volume displaced through the capillary tube. For gases at pressures less than 1,500 psia the corrected V_D was employed. ★★★